Letters to the Editors

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Tanner (1949), Katch (1972, 1973), and Katch and Katch (1974) and more recently Kronmal (1993), have criticised the use of a particular type of ratio, sometimes referred to as 'per ratio' standards, when studying the results of selected measurements in physiology and clinical medicine. In these disciplines, it is common practice to express various measurements (Y), such as oxygen consumption and cardiac output, as per-weight or per-surface area ratios, since by dividing by an "appropriate" body size variable (X), it is assumed that differences due to the subject's body size will have been removed, i.e. the ratio ($Y.X^{-1}$) is assumed to be independent of body size. Tanner observed that many of these 'per ratio' standards fail to render the measurements independent of body size, e.g. when maximum oxygen uptake ($l.min^{-1}$) is recorded per-body weight (kg), i.e. recorded in the usual weight-related units ($ml.kg^{-1}.min^{-1}$), and then correlated with body weight, the correlation is invariably found to be negative. Tanner and Katch (V.L.) proposed an alternative 'regression standard' to represent the subject's body-size adjusted measurement obtained by adding to the group mean, the subject's residual error taken from the linear regression on body size. Clearly, the assumptions associated with the use of regression standards as proposed by Tanner and Katch are, i) that the relationship between (Y) and (X) is linear, and ii) that the error term is additive with constant variance throughout the range of observations.

Kronmal (1993) recommended that a ratio should only be incorporated into regression analyses as part of a full linear model when the constituent parts that make up the ratio are also included as main effects, stating "it is not good practice to include interactions in an equations without first including the variables that comprise it as first order terms in the model". He supports his arguments with examples using Body Mass Index (BMI) and Forced Expiratory Volume (FEV). As with the regression standard models proposed by Tanner, Kronmal assumes that the models discussed in his article are linear with additive components and have error terms that are constant throughout the range of observations.

There can be little doubt that provided the above assumptions are acceptable, then the use of ratios in a linear model may indeed be misguided. However, there are an important class of models (1), often referred to as allometric or power function models, where the concept of a ratio is an integral part of the model form. A number of authors (e.g. Jolicoeur and Heusner (1971), Nevill et al. (1992), and Nevill and Holder (1994)) have adopted such models when investigating the relationship between various physiological response variables and the subjects' corresponding body size variable. For such relationships, the errors were found to be proportional to the dependent variable (heteroscedastic) and therefore multiplicative, as in (1),

$$Y=\alpha.X^{\beta}.\varepsilon. \qquad (1)$$

However, a log transformation of (1) gives a log-linear model that satisfies the usual additive requirements of linear regression. Confirmation of the log-linear model can be obtained using the methods of Box and Cox (1964) and Cox(1962).

When scaling various physiological measurements for individuals of different body size, Nevill et al. (1992) proposed the power function ratio standard ($Y_i.X_i^{-\beta}$), derived from the allometric model (1), to render the subject's physiological performance variable (Y_i) independent of their body size variable (X_i). The authors were able to confirm the findings of Astrand and Rodahl (1986) that maximum oxygen uptake was proportional to body mass, $m^{2/3}$ (i.e., oxygen consumption of a given muscle mass is thought to be approximately proportional to the cross-sectional area of the muscles used) and hence, should be scaled by recording maximum oxygen uptake in the units ($ml.kg^{-2/3}.min^{-1}$) to be independent of body size. They were also able to demonstrate that both peak and mean power output (W) needed to be scaled by recording peak and mean power in the unit ($W.kg^{-2/3}$) to be independent of body size.

Simply by taking logarithms of the power function ratio standard ($Y_i.X_i^{-\beta}$), Nevill et al. (1992) explained that the experimental design effects can be investigated using traditional ANOVA methods. The same design effects can be obtained using the analysis of covariance (ANCOVA), by analysing the $\log(Y_i)$ as the response (dependent) variable with the covariate $\log(X_i)$ automatically scaling the numerator dependent variable. Further covariates can be easily incorporated in the allometric model, for example Nevill and Holder (1994) incorporated age as a negative exponential term giving the additional operational advantage of providing a more plausible asymptotic decline with increasing age.

Since 'per ratio' standard variables are often proportional to each other, Nevill et al. (1992) used the multiplicative allometric model (2) to investigate the relationship between 5 kilometre run speed (Z), maximum oxygen uptake (Y) and body weight (X) of 204 recreationally active subjects (men n=112; women n=92),

$$Z(m.s^{-1})=\alpha.Y^{\beta_1}.X^{\beta_2}.\varepsilon \qquad (2)$$

Using multiple log-linear regression, no statistically significant differences were found between the male and female 'α' and 'β' parameters. The common fitted power function model relating 5 km running speed, $Z(m.s^{-1})$, to maximum oxygen uptake $Y(l.min^{-1})$ and body mass X(kg) was

$$Z(m.s^{-1})=84.3.Y^{1.01}.X^{-1.03}. \qquad (3)$$

Equation (3), is both simple and meaningful. The best predictor of 5 km run times, when recorded as a rate of performance, i.e. mean running speed ($m.s^{-1}$), is almost exactly proportional to the ratio standard maximum oxygen uptake ($l.min^{-1}$) divided by body mass (kg) or the well known weight-related maximum oxygen uptake units ($ml.kg^{-1}.min^{-1}$).

Following Kronmal (1993), the alternative full linear model to describe the running speed results Z, would be,

$$Z= \alpha+\beta_1.Y+\beta_2.X^{-1}+\beta_3.Y.X^{-1}+\varepsilon. \qquad (4)$$

In order to compare the allometric log linear model (2) with the equivalent full linear model (4), the criterion of Cox (1962) based on the difference between the two models' maximised log-likelihoods was used. The maximised log-likelihood statistic for the allometric model (2) was found to be -9.75 using just three parameters. In contrast, the maximised log-likelihood statistic for the full linear model (4) was less at -11.21 requiring four parameters, suggesting the superiority of the allometric model (2).

Since many of the variables used in physiology, clinical medicine and human performance are proportional to body size, these variables are better described with the allometric equation with a multiplicative error structure. Not only does the model form satisfy the requirement of proportionality together with a multiplicative error, but after a log transformation, the linear form complies with the modelling concerns raised by Kronmal, as described above.

Alan. M. Nevill, School of Sport and Exercise Sciences

Roger. L. Holder, School of Mathematics and Statistics

University of Birmingham

Edgbaston, Birmingham

B15 2TT, UK

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